Pass-Band VSWR of Maximally Flat Band-Pass Filters*

Microwave band-pass filters using high-Q cavity resonators can often be designed for almost negligible midband dissipation losses (*i.e.*, $<\frac{1}{2}$ db). If these filters are designed for a maximally flat amplitude response, the insertion loss within the pass band is approximately equal to the reflection loss within the pass band. Then

$$R = 10 \log (1 + X^{2n}),$$

where

R = reflection loss in db
X = normalized frequency variable
n = number of cavity resonators.

Letting ρ = input VSWR,

$$\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + X^n}{1 - X^n}.$$

Curves as ρ vs X have been plotted in Fig. 1 using the above equation for values of n from 1 to 8.

If the approximation of $X^{2n} \ll 1$ is not used, it can be shown that

$$\rho = \frac{2 + 4X^{2n} + \sqrt{(2 + 4X^{2n})^2 - 4}}{2}.$$

If $X^{2n} = 0.125$, $\rho = 2.0$ using the exact equation for ρ. If the approximate equation is used, with $X^{2n} = 0.125$ ρ is equal to 2.1. It can be concluded that the curves shown are satisfactory for most purposes when $\rho \leqq 2.0$.

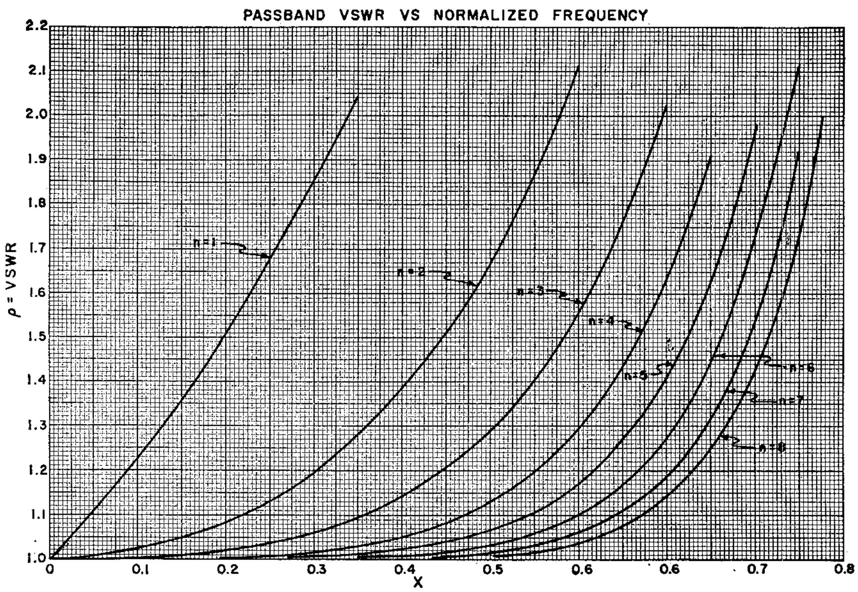

Fig. 1.

For narrow-band filters,

$$X \cong 2\left|\frac{f - f_0}{\Delta f}\right|,$$

where

f = frequency
f_0 = center frequency of filter
Δf = 3-db bandwidth of filter.

Now

$$1 + X^{2n} = \frac{1}{1 - |\Gamma|^2},$$

where

Γ = voltage reflection coefficient looking into the filter

$$1 - |\Gamma|^2 = \frac{1}{1 + X^{2n}} \cong 1 - X^{2n} \quad \text{if} \quad X^{2n} \ll 1$$

$$|\Gamma|^2 = X^{2n}$$

$$|\Gamma| = X^n.$$

The approximation used can also be applied to deriving equations for the VSWR of quasi-dissipationless filters of other response shapes, such as constant $-K$ or Tchebycheff.

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An Approximate Method of Finding the Order of a Combination Bessel-Function Equation*

Assume a transverse electric wave, TE mode, propagated in a waveguide with a circular boundary. The radial boundary condition gives a combination Bessel function equation; *i.e.*,

$$Z_p'(x_i) = J_p'(x)N_p'(x_i) - N_p'(x)J_p'(x_i) = 0, \quad (1)$$

where

$$x = \beta_c\rho_o \qquad x_i = \beta_c\rho_i.$$

The terms ρ_o and ρ_i are the radius of the circular boundaries of the waveguide, β_c is the cutoff wave number, and p is the order of the Bessel functions.

For a coaxial waveguide, p is determined from the angular boundary and is an integer such as 0, 1, 2, etc.

For a lunar line,[1] β_c is assumed and substituted in (1) to solve p. The arguments X and X_i are small, and the series form of the Bessel functions converges rapidly. By taking finite terms and using a graphic method, two real roots of p are obtained: one positive and one equal negative root. It is very complicated and difficult to find the complex roots using this method. However, using the following approximate method, the real roots and the complex roots p of (1) can be obtained. Eq. (1) can be written as

$$[J_p'(x_i)/J_p'(x)] = [N_p'(x_i)/N_p'(x)], \quad (2)$$

where

$$0 < (x_i - x) < 1.$$

By the Taylor series expansion,

$$J_p'(x_i) = J_p'(x) + (x_i - x)J_p''(x) + \frac{(x_i - x)^2}{2!}J_p'''(x) + \cdots. \quad (3)$$

Substituting the series forms of $J_p'(x_i)$ and $N_p'(x_i)$ into (2) simplifies that equation to

$$[J_p''(x)N_p'(x) - N_p''(x)J_p'(x)] + \frac{(x_i - x)}{2!}[J_p'''(x)N_p'(x) - N_p'''(x)J_p'(x)] + \frac{(x_i - x)^2}{3!}[J_p^{\text{IV}}(x)N_p'(x) - N_p^{\text{IV}}(x)J_p'(x)] + \frac{(x_i - x)^3}{4!}[J_p^{\text{V}}(x)N_p'(x) - N_p^{\text{V}}(x)J_p'(x)] + \cdots = 0. \quad (4)$$

From the differential equations,

$$J_p''(x) + \frac{1}{x}J_p'(x) + \left(1 - \frac{p^2}{x^2}\right)J_p(x) = 0 \quad (5)$$

and

$$N_p''(x) + \frac{1}{x}N_p'(x) + \left(1 - \frac{p^2}{x^2}\right)N_p(x) = 0. \quad (6)$$

The difference of (5) times $N_p'(x)$ and (6) times $J_p'(x)$ gives

$$J_p''(x)N_p'(x) - N_p''(x)J_p'(x) = \frac{2p^2}{\pi x^3} - \frac{2}{\pi x}. \quad (7)$$

* Received by the PGMTT, December 7, 1961.

[1] A. Y. Hu and A. Ishimaru, "The dominant cut-off wavelength of a lunar line," IRE Trans. on Microwave Theory and Techniques, vol. MTT-9, pp. 552–556; November, 1961.